Optimal parameter estimation in a landslide motion model using the adjoint method

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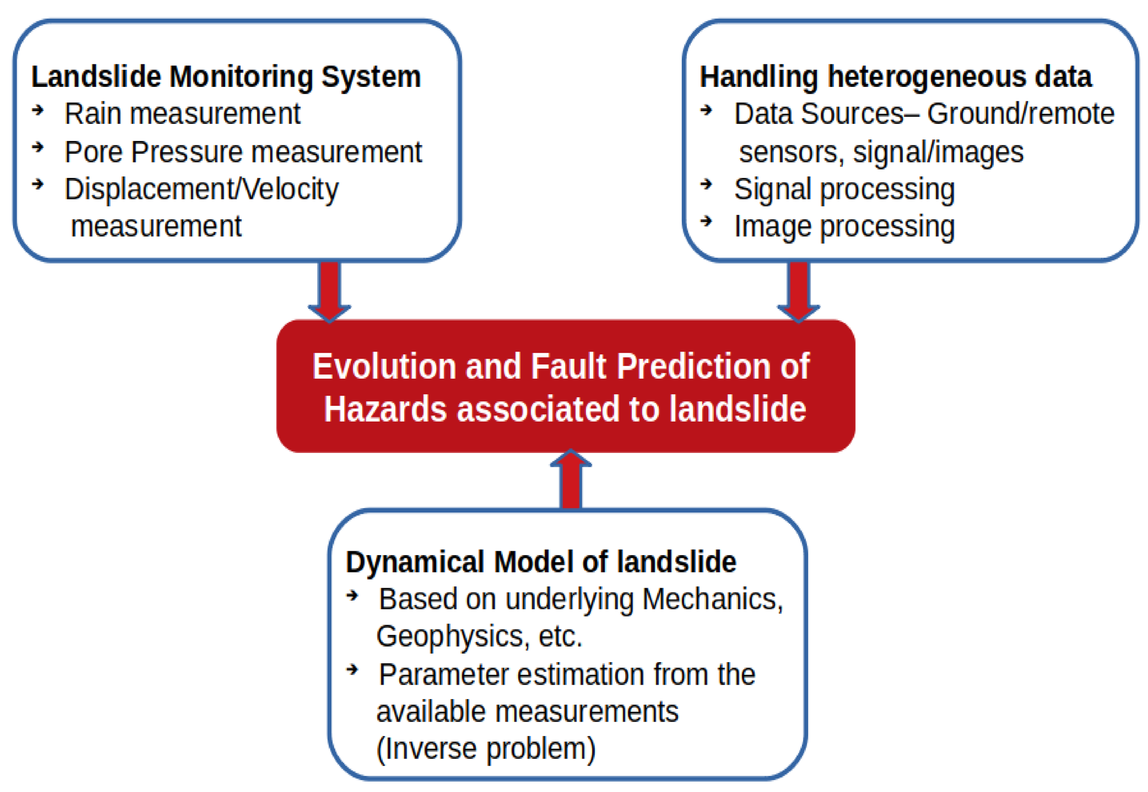
1. Objectives

1. **To estimate the mechanical strength and dilatancy angle of the material for the extended sliding-consolidation model (ODE-PDE coupled system) of a landslide**
2. **Proposing an optimal approach based on the adjoint method and steepest descent algorithm for parameter estimation**
3. **Validation of the proposed approach**

2. Context



5. Algorithm

Algorithm 1: Optimal parameter estimation

Input: Initial values, $v(0)$ & $p_e(z,0)$
Imposed pore pressure time series, $p_i(0,t)$
Measured velocity profile, $v_{mea}(t_k)$
Guessed parameter values, ϕ_G & ψ_G
Set initial parameter values, ϕ_0 & ψ_0
Step sizes, γ_ϕ & γ_ψ
The gradient tolerances, ξ_ϕ & ξ_ψ
Stop_flag = false
Iteration index k=1
Set $\phi^k = \phi_0$ and $\psi^k = \psi_0$
Output: ϕ^*, ψ^*
while *Stop_flag = false* **do**
  Simulate system equations with $v(0)$, $p_e(z,0)$, ϕ^k, & ψ^k;
  Simulate the adjoint system equations (backward in time);
  Compute gradients L_ϕ^k, L_ψ^k;
  if $|L_\phi| \le \xi_\phi$ & $|L_\psi| \le \xi_\psi$ **then**
    Stop_flag = true;
  else
    $\phi^{k+1} = \phi^k - \gamma_\phi L_\phi^k$;
    $\psi^{k+1} = \psi^k - \gamma_\psi L_\psi^k$;
    $k = k+1$;
  end
  Display ϕ, ψ and J
end
Return ϕ^*, ψ^*

6. Conclusion & future work

1. Adjoint method can be utilized for parameter estimation in landslide motion model.
2. The presented optimal estimation method will be validated ON REAL FIELD measurements.
3. This approach could be extended to more complex landslide models.

References

[1] Iverson R. M., *Regulation of landslide motion by dilatancy and pore-pressure feedback*, J. Physics. Res. - Earth Surface 110, No. F2, F02015, 2005.

[2] Van Tri Nguyen, Didier Georges, and Gildas Besançon, *State and parameter estimation in 1-D hyperbolic PDEs based on an adjoint method.* Automatica, 67, 185-191, 2016.

3. Extended sliding-consolidation model (dynamic model)

The model is based on Newton's second law where, landslide motion is opposed by basal Coulomb friction and regulated by basal pore fluid pressure [1].

Momentum equation

$$\frac{d^2u_x}{dt^2} = \frac{dv}{dt} = g\cos\psi\,[\sin(\theta-\psi) - \cos(\theta-\psi)\tan\phi] + \frac{\cos^2\psi\tan\phi}{\rho Z}[p_i(0,t)+p_e(0,t)] \qquad \text{i.e.}$$

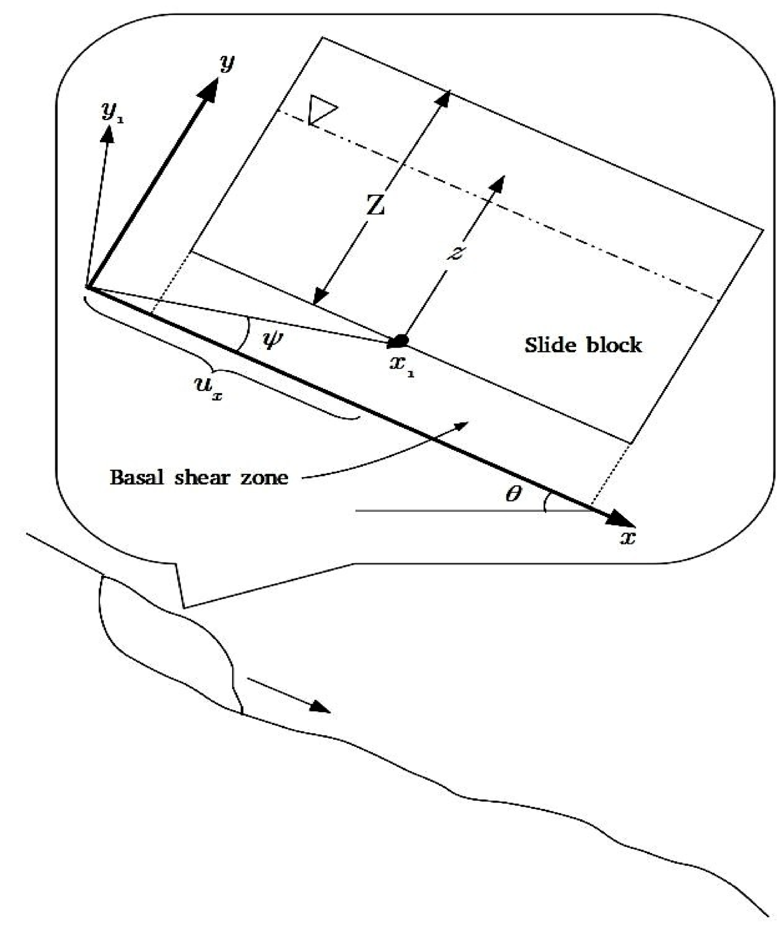
$$\dot{v} = f(\phi,\psi,p_e(0,t),p_i(0,t)), \qquad v(0)=v_0$$

Excess pore pressure diffusion equation

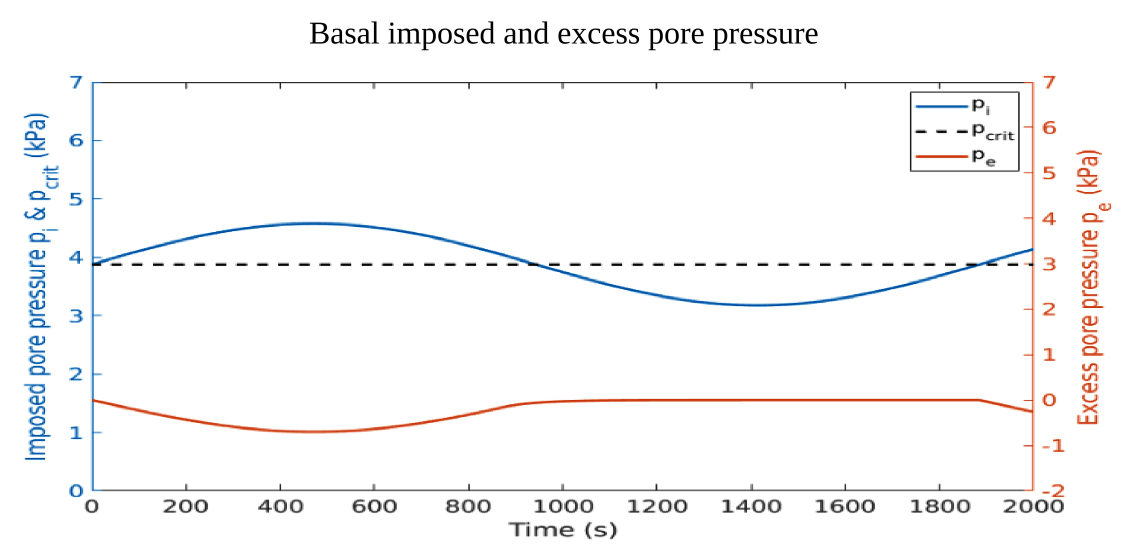
$$\frac{\partial p_e(z,t)}{\partial t} = D\frac{\partial^2 p_e(z,t)}{\partial z^2}$$

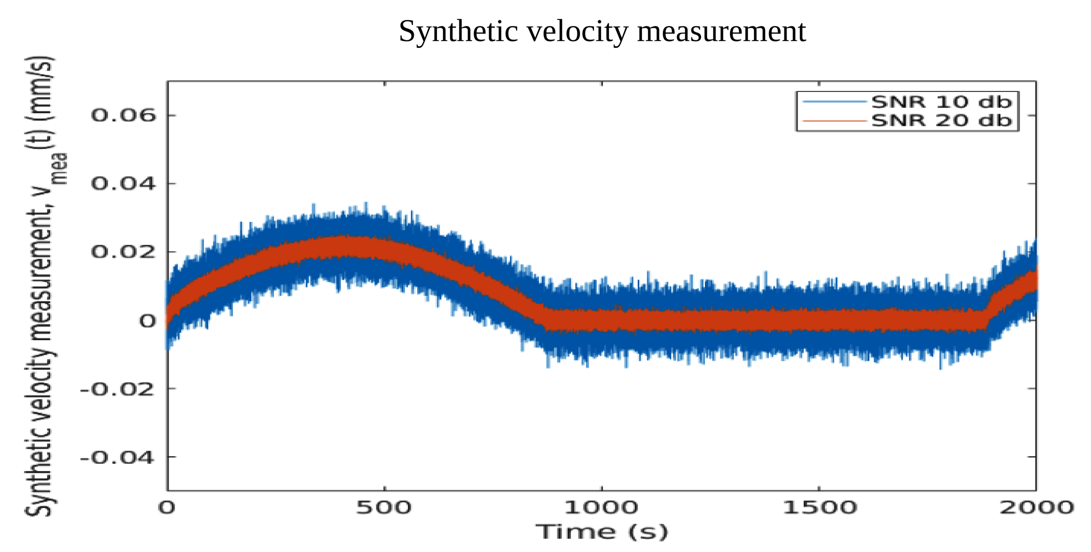
$$\frac{\partial p_e(0,t)}{\partial z} = \frac{\rho_w g\psi}{K}v,$$

$$p_e(Z,t)=0, \quad p_e(z,0)=p_{e_0}$$



Imposed pore pressure is assumed to be sinusoidal (representing rainfall variations)





4. Parameter estimation by adjoint method

Adjoint Method: Iterative numerical method for efficiently computing the gradiants of a cost functional in a numerical (constraint) optimization problem [2]. Here we are interested in **estimation of ψ & ϕ from available velocity measurements** $v_{mea}(t_k)$. Cost function (J) & Cost functional (L) are defined as follows:

$$J(\phi,\psi) = \frac{\varepsilon_1}{2}\|\phi-\phi_G\|^2 + \frac{\varepsilon_2}{2}\|\psi-\psi_G\|^2 + \frac{\varepsilon_3}{2}\sum_{k=1}^{N}\left(\int_0^T \delta_A(t-t_k)v(t)dt - v_{mea}(t_k)\right)^2$$

$$L(v,p_e,\phi,\psi) = J + \int_0^T \lambda(t)\,[\dot{v} - f(\psi,\phi,p_e(0,t),p_i(0,t))]\,dt + \int_0^T\int_0^Z \Gamma(z,t)\left(\frac{\partial p_e}{\partial t} - D\frac{\partial^2 p_e}{\partial z^2}\right)dz\,dt$$

- Adjoint system equations:

$$L_v = 0 \;\Leftrightarrow\; \begin{cases} \dot{\lambda} = \varepsilon_3\sum_{k=1}^{N}\delta_A(t-t_k)\left[\int_0^T \delta_A(t-t_k)v(t)dt - v_{mea}(t_k)\right] + \frac{D\rho_w g\psi}{K}\Gamma(0,t) \\ \lambda(T)=0 \end{cases} \quad \& \quad L_{p_e}=0 \;\Leftrightarrow\; \begin{cases} \frac{\partial\Gamma(z,t)}{\partial t} = -D\frac{\partial^2(z,t)\Gamma}{\partial z^2} \\ \frac{\partial\Gamma(0,t)}{\partial z} = -\frac{1}{D}\lambda(t)f_{p_e(0,t)} \\ \Gamma(Z,t)=0 \\ \Gamma(z,T)=0 \end{cases}$$

- Gradients:

$$L_\phi = \frac{\partial L}{\partial\phi} = \varepsilon_1(\phi-\phi_G) - \int_0^T \lambda(t)f_\phi(t)dt \quad \& \quad L_\psi = \frac{\partial L}{\partial\psi} = \varepsilon_2(\psi-\psi_G) - \int_0^T \lambda(t)f_\psi(t)dt + \frac{D\rho_w g}{K}\int_0^T \Gamma(0,t)v(t)dt$$